4/MTH–252 Syllabus–2023

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(May–June)

FYUP : 4th Semester Examination

MATHEMATICS

(Dynamics—I)

(MTH–252)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **four** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* A particle is moving with SHM and while making an excursion from one position of rest to the other, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3. Prove that the time of a complete oscillation is

$$\frac{2\pi}{\cos^{-1}\left(\dfrac{x_1+x_3}{2x_2}\right)}$$ 6

(b) A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes to reach a height h is

$$\frac{1}{3}\sqrt{\frac{2r}{g}}\left[\left(1+\frac{h}{r}\right)^{\frac{3}{2}}-1\right]$$ 6

(c) A particle moves in a straight line, starting from rest at a distance a, towards the centre of force. If its acceleration at a distance x from the centre of force be $\frac{\mu}{x^{\frac{5}{3}}}$, show that it will reach the origin after a time $\frac{2a^{\frac{4}{3}}}{\sqrt{3\mu}}$. 6

2. (a) A particle moving in a straight line is subject to a resistance which produces a retardation kv^3, where v is the velocity and k a constant. Show that v and t are given in terms of s (the distance) by the equations $v=\frac{u}{1+ksu}$, $t=\frac{s}{u}+\frac{1}{2}ks^2$, where u is its initial velocity. Also show that

$$s=\frac{1}{ku}[\sqrt{1+2ku^2t}-1] \text{ and } v=\frac{u}{\sqrt{1+2ku^2t}}$$ 6

(b) A particle, of mass m, is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}[\lambda-\log(1+\lambda)]$, where V is the terminal velocity of the particle and λV is its initial vertical velocity. Show that the corresponding time is

$$\frac{V}{g}\log(1+\lambda)$$ 6

(c) A particle moves along the axis of x starting from rest at $x=a$. For an interval t_1 from the beginning of motion the acceleration is $-\mu x$, for a subsequent time t_2, the acceleration is μx and at the end of this interval the particle is at the origin. Prove that

$$\tan(\sqrt{\mu}t_1)\cdot\tanh(\sqrt{\mu}t_2)=1$$ 6

UNIT—II

3. (a) The coordinates (x, y) of a moving point at time t are given by $x=a(2t+\sin 2t)$, $y=a(1-\cos 2t)$. Prove that its acceleration is constant. 2

(b) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$. Find the path and show that the accelerations along and perpendicular to the radius vector are

$$\lambda^2 r - \frac{\mu^2\theta^2}{r} \text{ and } \mu\theta\left(\lambda + \frac{\mu}{r}\right)$$ 5

(c) If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity. 3

(d) A point describes the cycloid $s = 4a\sin\psi$ with uniform speed u. Find its acceleration at any point in terms of u, a and s. 4

(e) A particle describes a curve (for which S and ψ vanish simultaneously) with uniform speed u. If the acceleration at any point s is $\frac{u^2 c}{s^2 + c^2}$, find the intrinsic equation of the curve. 5

4. (a) If any point of the parabolic path the velocity be u and the inclination to the horizon be θ, show that the particle is moving at right angles to its former direction after a time $\frac{u}{g}\operatorname{cosec}\theta$. 6

(b) A body is projected at an angle α to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to $2a\cot\frac{\alpha}{2}$. 6

(c) If t be the time in which a projectile reaches a point P in its path and t' the time from P till it reaches the horizontal plane through the point of projection. Show that the height of P above the horizontal plane is $\frac{1}{2}gtt'$. Also prove that the greatest height of the projectile is

$$\frac{1}{8}g(t+t')^2$$ 7

UNIT—III

5. (a) A shell lying in a straight smooth horizontal tube suddenly breaks into two portions of masses m_1 and m_2. If s is the distance apart in the tube of the masses after time t, show that the work done by the explosion is

$$\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}\frac{s^2}{t^2}$$ 7

(b) A shot of mass m is fired from a gun of mass M with a velocity u relative to the gun. Show that the actual velocities of the shot and the gun are

$$\frac{Mu}{M+m} \text{ and } \frac{mu}{M+m}$$

respectively and their kinetic energies are inversely proportional to their masses. 7

(c) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Prove that the velocity of the recoil is

$$\sqrt{\frac{2m^2gh}{M(M+m)}}$$ 5

6. (a) A ball A of mass m_1 impinges directly on another ball B of mass m_2, which is at rest. After the impact B impinges directly on a third ball of mass m_3 which is also at rest. If the velocity imparted to C is same as A had at first, and if all the balls are perfectly elastic, show that $(m_1+m_2)(m_2+m_3)=4m_1m_2$. 7

(b) A sphere m_1 impinges obliquely on another sphere m_2 which is at rest. If $m_1 = em_2$, show that they will move at right angles to each other. 7

(c) A heavy elastic ball drops from the ceiling of a room and after rebounding twice from the floor reaches a height equal to one half of that of the ceiling. Show that $2e^4 = 1$. 5

UNIT—IV

7. (a) A particle moves on the outside of the arc of a smooth vertical circle. If it starts from rest down the arc from a point A, show that it will leave the circle at B, where $\theta = \cos^{-1}\left(\frac{2}{3}\cos\alpha\right)$, θ and α being the arcual distances of B and A respectively from the highest point. 7

(b) A particle projected along the inside of a smooth vertical circle of radius a from the lowest point. Show that the velocity of projection required in order that after leaving the circle the particle may pass through the centre is $\sqrt{\frac{1}{2}ag(\sqrt{3}+1)}$. 7

(c) A particle moves on the inside of a vertical circle and just reaches one end of the horizontal diameter. Show that the reaction at any point is proportional to the depth below the horizontal diameter. 5

8. *(a)* A heavy particle slides down a smooth cycloid, starting from rest at a cusp, the axis being vertical and vertex downwards. Prove that the magnitude of the acceleration is equal to g at every point of the path. 6

(b) A particle oscillates in a cycloid under gravity. The amplitude of the motion being b and the periodic time T. Show that its velocity at a time measured from the position of rest is

$$\frac{2\pi b}{T}\sin\left(\frac{2\pi t}{T}\right)$$ 7

(c) A particle slides from rest down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down to the second half. 6

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